

VARIANCE GAMMA MODEL AND ITS DEVELOPMENT FOR STOCKS CALL OPTION PRICES ESTIMATION

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ABSTRACT

One of the developments in the options market is the formation of various pricing models for an option to help the buyer determine the fairness of the price. Black-Scholes model uses the assumption that the log return price of stocks is normally distributed, while in reality, the real-world price couldn't fit into that assumption. To be able to obtain an option price calculation that considers skewness and kurtosis in the stock price data, there are many alternative methods, namely Black-Scholes with Gram-Charlier expansion and Variance Gamma models. As a development of the Variance Gamma model, there are also several methods that are able to reduce the simulations variance generated by the Variance Gamma model, namely Antithetic Variate Variance Gamma model and the Importance Sampling Variance Gamma method. For the results, the Importance Sampling Variance Gamma model and the Antithetic Variate Variance Gamma model are really able to reduce the resulting simulations variance so that it can produce more accurate option prices with market prices compared to the Variance Gamma model. In the end, all Variance Gamma models are able to produce a call option price that is more in line with the market price than all Black Scholes models.

Keywords: Black-Scholes, Call Option, Gram Charlier Expansion Black Scholes, Variance-Gamma, Variance Reduction.

INTRODUCTION

An option is the right to buy or sell securities or commodities at an agreed price at a specific price during the contract period (Smith, 1976). It can be an alternative to buying securities that have high risk so that investors can buy them at relatively low prices and can control the risk (Susanto and Malelak, 2021).

One of the developments in the options market is the formation of various pricing models for an option. The model can help the buyer of an option to determine the fairness of an option price. One of the commonly used option pricing methods is the Black Scholes Model. The Black-Scholes model is a mathematical model in the form of a partial differential equation and is used in determining the price of European options (Desmon, 2004). This model uses the assumption that the log return price of stocks is normally distributed. However, this assumption cannot be fully met in empirical data, where empirical data often have skewness and kurtosis. As a result of this, the prices obtained with the Black Scholes Model are often inconsistent with market prices.

To be able to obtain an option price calculation that considers skewness and kurtosis, there are many alternative methods. One model that can be an alternative to the Black Scholes model is the Black-Scholes with Gram-Charlier expansion and Variance Gamma models. The Gram-Charlier Expansion method is used to include option price adjustments for abnormal skewness and kurtosis in the Black-Scholes formula (Corrado and Su, 1996). On the other hand, the Variance Gamma model was introduced as a development of Geometric Brownian Motion to overcome the skewness and kurtosis of the stock price return distribution. As a development of the Variance Gamma model, there are also several methods that are able to reduce the variance value of the call_price simulation generated by the Variance Gamma model. These models are the Antithetic Variance Reduction Variance Gamma (AVR VG) or Antithetic Variate Variance Gamma model and the Importance Sampling Variance Gamma method.

LITERATURE REVIEW

The Black-Scholes model is a model developed by Fischer Black and Myron Scholes to determine option prices that has been widely accepted by the financial community. According to Desmon

(2004), there are several assumptions used in the model, namely the option used is a European type option (European option), the stock price variance is constant over the life of the option and is known with certainty, a random process in obtaining stock prices, and central interest rates. Although Black-Scholes is good to determine option prices, Black-Scholes model uses the assumption that the log return price of stocks is normally distributed, while in reality, the real-world price could not fit into that assumption.

Based on Chateau and Dufresne (2017), Gram-Charlier is a distribution with a density in the form of a polynomial multiplied by a normal density. In determining the price of options using this distribution, it will maintain the nature of the normal distribution by also considering the non-zero skewness and kurtosis. As the results, the Gram Charlier Expansion Black Scholes formula can really adjust the abnormal skewness and kurtosis in the option pricing using the Black-Scholes formula.

Based on Bolia and Juneja (2005), Monte Carlo simulation is a popular computational tool to calculate the complex financial option price. This method evaluates the expectation of a random variable by generating many random variable samples that are independent and takes the empirical mean from those samples as the point estimation of an expectation. This method accuracy is proportional to σ/\sqrt{n} where σ^2 is the variance of each sample and n is the number of samples generated. The advantages of this model are that the computational effort to reach an intended accuracy is independent with the problem's dimension. So, for the option with many dimensions in the asset, this method gives a promising approach.

METHODOLOGY

In this paper, we will estimate the value of stock call option prices using several methods, namely the Black-Scholes method and its development method, namely Black-Scholes with Gram-Charlier expansion, and the Variance-Gamma method and its development method, namely Antithetic Variate Variance Gamma model and the Importance Sampling Variance Gamma method. The dataset used in this analysis is the daily stock data of NIO Inc. from the automotive sector and the daily stock of Intel Corporation from the technology sector. Stock data was taken from May 21, 2020 to May 20, 2021 from finance.yahoo.com. The price of the option being compared is a call option with a period of 1 month for June 25, 2021 for the NIO stock, and for July 2, 2021 for the INTC stock. The interest rate used is the interest rate of the United States Bank Daily Treasury Bill Rate obtained from treasury.gov, which is 0.04. The variance gamma method and some of its development methods will be discussed in the following subsections.

Variance Gamma Method

Based on Madan et al. (1998), the variance gamma process is obtained by evaluating the Brownian motion in random time change with the gamma process. In this process, the return time unit which is continuously combined is normally distributed, depending on the random time realization. This random time has a gamma density. The result of the stochastic process and the option pricing model that is obtained would result in 3 robust parameters. These parameters would give control to the kurtosis, which is the symmetrical increase in the left and right tail of the return distribution; and skewness, which gives the asymmetrical shape to the left and right tail of return density.

Variance gamma process is defined with respect to the Brownian motion with drift $b(t; \theta, \sigma)$ and the gamma process with unit mean rate $(\gamma(t; 1, \nu))$ as:

$$X(t; \sigma, \nu, \theta) = b(\gamma(t; 1, \nu); \theta, \sigma) \dots\dots\dots(1)$$

That variance gamma process is obtained by evaluating the Brownian motion at a certain time given the gamma process. This variance gamma process has 3 parameters, σ = the volatility of Brownian motion, ν = variance rate of the gamma time change, and θ = drift in the Brownian motion with drift. This process give 2 dimension control to the distribution, that is to the skewness with the parameter θ , and to the kurtosis with the parameter ν .

$X(t)$ at time interval t is a variance gamma random variable with the normal distribution could be shown as:

$$X_{VG}(t) = \theta g + \sigma\sqrt{gz} \dots\dots\dots(2)$$

Where z is the standard normal distribution random variable that is independent with g , which is the gamma distribution random variable with mean t and variance vt . This variance gamma random variable has mean θg and variance $\sigma\sqrt{g}$.

Variance gamma parameters could be estimated with the moment method. The parameters could be estimated with:

1. $\hat{\sigma} = \sqrt{Var(X)}$
2. $\hat{\theta} = \frac{\sigma Skewness(X)}{3v}$
3. $\hat{v} = \frac{Kurtosis(X)}{3} - 1$

The option pricing model with variance gamma could be calculated as:

$$S(t) = S(0)exp[(r + \omega)t + X_{VG}(t)] \dots\dots\dots(3)$$

where $\omega = \frac{1}{v} \ln(1 - \theta v - \frac{1}{2} \sigma^2 v)$

and $X_{VG}(t) = \theta g + \sigma\sqrt{gz}$,

which is the variance gamma process obtained from the standard normal process of gamma process.

Antithetic Variance Reduction Simulations

From Bolia and Juneja (2005), antithetic variance method has the base idea of considering 2 continuous random variables X_1 and X_2 with the variance of the sums as:

$$\sigma_{X_1+X_2}^2 = \sigma_{X_1}^2 + \sigma_{X_2}^2 + 2\rho_{XY}\sigma_{X_1}\sigma_{X_2} \dots\dots\dots(4)$$

If both of the random variables are negatively correlated, then the variance of the sums will be less than the variance when both of them are independent. If X_1 and X_2 have the distribution function $F_1(\cdot)$ and $F_2(\cdot)$, then if U is uniformly distributed, $F_1^{-1}(U)$ will have the distribution of X_1 . $F_1^{-1}(U)$ and $F_2^{-1}(1 - U)$ will result in highest negative correlation possible between the distributions with marginal distribution of X_1 dan X_2 . If N is the sample of Gaussian random variable with mean 0 and variance σ^2 , then $-N$ would also have the same distribution and have the correlation of -1 with N . This Antithetic Variate technique takes the antithetic path for each sample path. That is for example if there is a path $\varepsilon_1, \varepsilon_2, \dots, \varepsilon_M$ then the path $-\varepsilon_1, -\varepsilon_2, \dots, -\varepsilon_M$ would also be taken.

From Sigman (2007), it is used:

$$\bar{X}(n) = \frac{1}{2m} \sum_{j=1}^{2m} X_j = \frac{1}{m} \sum_{j=1}^m Y_j = \bar{Y}(m) \dots\dots\dots(5)$$

where both of the estimations are identical. Next, $E(Y_i) = E(X) = \mu$, so Y_i could be seen as the copy that would be simulated from X . Because of that, it is formed $Y = \frac{X_1+X_2}{2}$ as the generalization of Y_i and the variance value:

$$Var(Y) = \frac{1}{4}(\sigma^2 + \sigma^2 + 2Cov(X_1, X_2)) = \frac{1}{2}(\sigma^2 + Cov(X_1, X_2)) \dots\dots\dots(6)$$

In the case where X_i iid, $Cov(X_1, X_2) = 0$ then $Var(Y) = \sigma^2/2$ and when $Cov(X_1, X_2) < 0$ then $Var(Y) < \sigma^2/2$, where the variance would be reduced.

Importance Sampling Simulations

This Importance Sampling Method has the base idea by concentrating the simulation on the sample path that gives the highest contribution in the payment estimation (Su, Yi, et.al, 2002). Importance sampling estimates the expectation of a random variable by generating sample paths with a probability measure that is different from the original one. This is done by prioritizing the important path of the estimation from different point of view. The estimation obtained would be made unbiased by using likelihood ratio or Radon-Nikodym derivative. If it is done correctly, this model would yield a much smaller variance (Bolia, N, et.al, 2005).

ANALYSIS AND DISCUSSION

Analysis

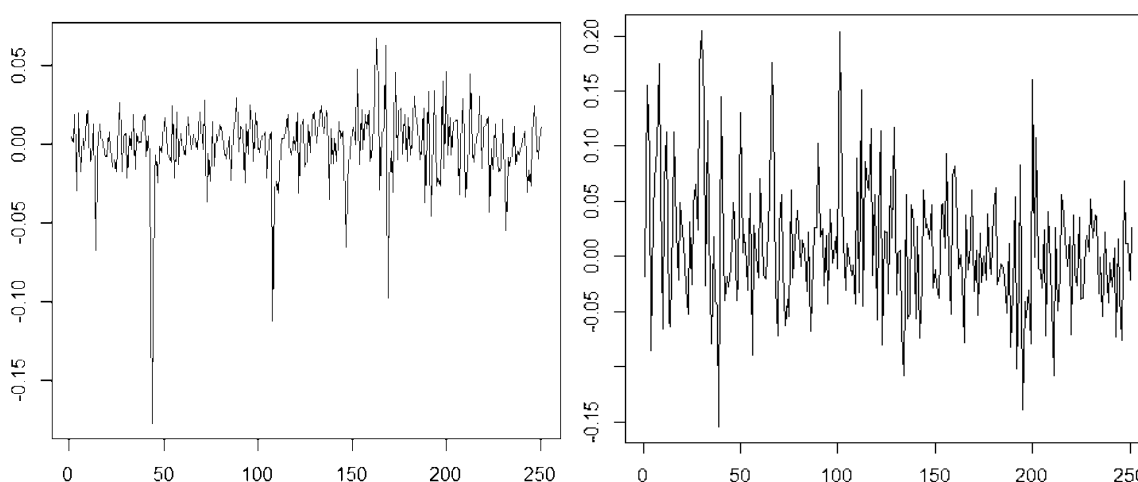


Figure 1. Log return of INTC (left) and NIO (right).

Table 1. The Characteristics of the Stocks.

Stock	Mean	Variance	Jarque Bera
INTC	-0.0004077821	0.1469788	0.00000000000000022
NIO	0.009295009	0.8870534	0.000003594

From Figure 1, we can see that INTC's log return volatility tends to be more stable throughout the year even though it has many dropdowns when compared to the NIO stock's log return volatility. On the other hand, from Table 1, it can be seen the normality of the return data which is time series data can be obtained by the Jarque Bera test. With the null hypothesis that the log return data is normally distributed, the significance level of alpha is 0.05, and the critical area we have is the null hypothesis is rejected if p-value is less than alpha, we get p-values for Intel Corp. stocks and NIO stocks of 0.00000000000000022 and 0.000003594, respectively. Because both p-values are less than alpha, the null hypothesis is rejected, which means that both of the log return data are not normally distributed. Because it is found that the return data are not normally distributed for the two stocks, the skewness and kurtosis of each stock will be seen next.

Table 2. Skewness and Kurtosis for each Stocks.

Stock	Skewness	Kurtosis
INTC	-2.174692	16.15752
NIO	0.6279755	3.905527

From Table 2, we can see clearly that INTC stock's log return has high kurtosis values (>3), which confirms that it has heavier tails than implied by the normal distribution. Because the coefficient of kurtosis is greater than 3, we can say that the data distribution is leptokurtic and shows a sharp peak on the graph (GeeksforGeeks, 2020). This also applies to NIO stocks where the kurtosis value is so high which confirms that it has heavier tails than implied by the normal distribution. On the other hand, we can see that the coefficient of skewness of INTC stock's log return is less than 0, so it means the graph is said to be negatively skewed with the majority of data values greater than mean. Different with AAPL stock, the coefficient of skewness of NIO stock's log return is greater than 0, so it means the graph is said to be positively skewed with the majority of data values less than mean. Most of the values are concentrated on the left side of the graph.

Table 3. Variance Gamma Parameters Estimation.

Stock	Sigma	Theta	Nu
INTC	0.021184	-0.007613	0.766856
NIO	0.05109	0.07916	0.12099

Table 4. Goodness-of-Fit with Kolmogorov-Smirnov test for Variance Gamma distribution.

Stock	D	Critical Value
INTC	0.05430662	0.08584244
NIO	0.04080745	0.08584244

To overcome the skewness and kurtosis of the distribution of each stock price return, we would fit the variance-gamma distribution. The parameters of the gamma-variance distribution are obtained as written in Table 3. On the other hand, in the Goodness-of-Fit with Kolmogorov-Smirnov test for Variance Gamma distribution that is written in Table 4, with the null hypothesis that the log return data is really from Variance Gamma distribution, the significance level of alpha is 0.05, and the critical area we have is the null hypothesis is rejected if D is larger than the critical value, we get D for Intel Corp. stocks and NIO stocks of 0.05430662 and 0.04080745, respectively. Because both D are less than the critical value, the null hypothesis is not rejected, which means that both of the log return data are really from Variance Gamma distribution.

Table 5. Call Option Price Estimation using B-S Model.

Stock	S0	K	B-S Price	Market Price	MAPE
INTC	55.95	53	5.33237284	3.22	65.60164%
NIO	27.0	34.06	8.7710423	7.83	12.01842%

The call option price is calculated for INTC stock and NIO stock using the OptionPricing library for the Black Scholes option. INTC stock options are used with strike price (K) 53 and strike price (K) 34.06 for NIO stock. We can see from the table above that the MAPE for INTC and NIO stocks is 65.60164% and 12.01842%, respectively, so we can say they are quite large. This shows that the actual option selling price and the estimated option selling price using the Black Scholes formula are a bit different. In addition, because the MAPE of NIO stock is smaller than that of INTC's stock, it can be concluded that based on the Black Scholes model, NIO stock has a more reasonable option price when compared to INTC stock.

Table 6. Call Option Price Estimation using Charlier Expansion B-S Model.

Stock	S0	K	B-S Price	Market Price	MAPE
INTC	55.95	53	3.952221	3.22	22.73977%
NIO	27.0	34.06	8.510307	7.83	8.68847%

We can see from the table above that the Mean Absolute Percentage Error for INTC and NIO is 22.73977% and 8.68847%, respectively, so we can say it's quite large for INTC and small for NIO. This shows that the Mean Absolute Percentage Error for INTC and NIO resulting from using the Gram Charlier Expansion Black Scholes formula are lower than using the Black Scholes formula. It means that the Gram Charlier Expansion Black Scholes formula can really adjust option price with

abnormal skewness and kurtosis in the Black-Scholes formula. But still, the actual option selling price and the estimated option selling price using the Gram Charlier Expansion Black Scholes formula are a bit different for INTC, although they are not significantly different for NIO. In addition, because the Mean Absolute Percentage Error of NIO's stock is smaller than that of INTC's, it can be concluded that based on the Gram Charlier Expansion Black Scholes model, NIO's stock have a more reasonable option price when compared to INTC's stock.

Table 7. Standard Error Comparison for INTC Stock.

nSim	Variance Gamma		Antithetic Variate Variance Gamma		Importance Sampling Variance Gamma	
	VG Price	SE	AVR VG Price	SE	IS VG Price	SE
5	3.160012	0.228471	3.315790	0.141901	3.483736	0.173202
10	2.970352	0.162886	2.865754	0.134981	3.096101	0.138093
20	3.297964	0.111657	3.235684	0.107981	3.085029	0.161993
25	3.206247	0.070934	3.272595	0.065799	3.399266	0.086607
50	3.271191	0.045463	3.270172	0.035712	2.950961	0.116665
500	3.298284	0.022635	3.304075	0.017330	3.277514	0.002782
5000	3.320694	0.007127	3.311554	0.005102	3.396376	0.013697
50000	3.309931	0.002299	3.307001	0.001631	3.370353	0.000001
500000	3.310818	0.000726	3.310703	0.000512	3.384396	0.001196
5000000	3.310897	0.000230	3.310667	0.000162	3.382462	0.000191

Simulations of option prices for INTC stock are carried out using Variance Gamma, Antithetic Variate Variance Gamma, and Importance Sampling Variance Gamma models with 10 values of n. The INTC option is used with a strike price (K) of 53. We can see from the table above that the more simulations we do, the smaller the standard errors we get, both for the Variance Gamma model, Antithetic Variate Variance Gamma, and Importance Sampling Variance Gamma for INTC stock option prices. On the other hand, we can see that the standard errors generated by the Importance Sampling Variance Gamma model are sometimes smaller than the other two models and the Antithetic Variate Variance Gamma's are always smaller than the Variance Gamma's model for any number of simulations. From this, it can be concluded that the fluctuation level of the Importance Sampling Variance Gamma model and Antithetic Variate Variance Gamma model is lower than the Variance Gamma model and the fluctuation level of the Antithetic Variate Variance Gamma model is lower than the fluctuation level of the Variance Gamma model. Based on this, in this simulation, for the price of INTC stock options, it is proven that the Importance Sampling Variance Gamma model and the Antithetic Variate Variance Gamma model are actually able to reduce the resulting simulation variance so that it can produce more accurate option prices with market prices compared to the Variance-Gamma Model.

Table 8. Standard Error Comparison for NIO Stock.

nSim	Variance Gamma		Antithetic Variate Variance Gamma		Importance Sampling Variance Gamma	
	VG Price	SE	AVR VG Price	SE	IS VG Price	SE
5	7.102317	0.241103	7.057001	0.099207	7.242386	0.250845
10	7.18974	0.317898	7.251891	0.28664	6.676198	0.140514
20	7.257384	0.141	7.2047	0.103846	6.880311	0.09282
25	7.126214	0.144262	7.137008	0.104621	7.174313	0.016952
50	7.178581	0.108792	7.198726	0.06554	6.721746	0.12035
500	7.160842	0.033633	7.208541	0.023419	7.248107	0.020796
5000	7.234664	0.010229	7.222642	0.007299	7.543122	0.004638
50000	7.205628	0.003307	7.206966	0.002338	7.033529	0.000929
500000	7.207991	0.001044	7.208828	0.000738	7.073266	0.001393
5000000	7.209008	0.00033	7.208998	0.000234	7.039181	0.000172

Simulations of option prices for NIO stock are carried out using the Variance Gamma, Antithetic Variate Variance Gamma, and Importance Sampling Variance Gamma models with 10 values of n. The NIO option is used with a strike price (K) of 27. We can see from the table above that in

general, the more simulations we do, the smaller the standard errors we get, whether for the Variance Gamma model, Antithetic Variate Variance Gamma, as well as Importance Sampling Variance Gamma for NIO stock option prices. On the other hand, we can see that the standard errors generated by the Importance Sampling Variance Gamma model are very often smaller than the other two models and the Antithetic Variate Variance Gamma's are always smaller than the Variance Gamma's model (sometimes even smaller than the Importance Sampling Variance Gamma's model) for any number of simulations. From this, it can be concluded that in general the fluctuation level of the Importance Sampling Variance Gamma model is lower than the other two models and the fluctuation level of the Antithetic Variate Variance Gamma model is lower than the fluctuation level of the Variance Gamma model. Based on this, in these simulations, for the NIO stock option price, it is proven that the Importance Sampling Variance Gamma model and the Antithetic Variate Variance Gamma model are actually able to reduce the resulting simulations variance so that it can produce more accurate option prices with market prices compared to the Variance-Gamma model.

Now, we will compare the Mean Absolute Percentage Error (MAPE) of call option price estimation made by Black Scholes Model, Gram-Charlier Expansion Black Scholes Model, Variance Gamma Model, Antithetic Variate Variance Gamma Model, and Importance Sampling Variance Gamma Model to find out which model is able to estimate the call price that is close to the real market price. The real market price of INTC stock is 3.22 and the real market price of NIO stock is 7.83. The comparison can be seen in table 9.

Table 9. MAPE Comparison.

Stock	BS	BS GC	VG	VG AVR	VG IS
INTC	65.60164%	22.73977%	2.804872%	3.14437%	5.223645%
NIO	12.01842%	8.68847%	7.83446%	7.887012%	7.573566%

Overall, the MAPE of all Variance Gamma models is much lower than that of all Black-Scholes models, both for INTC stocks and NIO stocks. This shows that all Variance Gamma models are able to produce a call option price that is more in line with the market price than all Black Scholes models. This is reinforced by stock characteristics where each stock return log is not normally distributed, and that Variance Gamma model could accommodate those characteristics better than the Black Scholes model. Moreover, the probability density function implied by the Variance Gamma model provides a better fit than the normal distribution, so that in this case, of course, all Variance Gamma models will be better than all Black Scholes models.

Discussion

We first compute the option pricing with the Black Scholes method and the Gram Charlier Expansion method. We then found that the Mean Absolute Percentage Error for INTC and NIO resulting from using the Gram Charlier Expansion Black Scholes formula are lower than using the Black Scholes formula. This means that the Gram Charlier Expansion Black Scholes formula can really adjust the abnormal skewness and kurtosis in the option pricing using the Black-Scholes formula.

Next, we compute the option pricing of both stock prices using the Variance Gamma model and the variance reduction simulations models that are Antithetic Variate Variance Gamma and Important Sampling Variance Gamma. Whether using Variance Gamma, Antithetic Variate Variance Gamma, and Importance Sampling Variance Gamma models, in general, we get the result that the standard errors we get will be smaller as the number of simulations increases. On the other hand, for both stocks, the fluctuation level of the Importance Sampling Variance Gamma model is generally lower than the other two models and the fluctuation level of the Antithetic Variate Variance Gamma model is lower than the fluctuation level of the Variance Gamma model. So, in this simulation, the Importance Sampling Variance Gamma model and the Antithetic Variate Variance Gamma model are really able to reduce the resulting simulations variance so that it can produce more accurate option prices with market prices compared to the Variance Gamma model.

Finally, we would like to compare each price from the models that we have simulated before to the market price. The result we obtained is that the MAPE from the Variance Gamma models, whether it is the original one or the variance reduction one, is always smaller than the Black Scholes models, whether it is the original one or the Gram Charlier Expansion one. This shows that all Variance Gamma models are able to produce a call option price that is more in line with the market price than all Black Scholes models. This is reinforced by stock characteristics where each stock log return is not normally distributed, and that Variance Gamma model could accommodate those characteristics better than the Black Scholes model. More specifically, we can see that the best model that gives the closest price to the real market price seen by the MAPE value on NIO stock is the Importance Sampling Variance Gamma model. Theoretically, this makes sense because the Importance Sampling Variance Gamma model could reduce the simulations' standard errors from the original Variance Gamma simulations model. On the other hand, the best model that gives the closest price to the real market price on INTC stock is the original Variance Gamma model. This is quite different from the theory that the variance reduction models could give a more accurate price. This could happen because the price volatility in INTC stock tends to be stable throughout the year which means it has a small variance. The results of the reduction of variance in INTC where the variance is already low tend to make the Importance Sampling Variance Gamma model too generalized compared to the original Variance Gamma model, so that the original Variance Gamma model is more fit than the Importance Sampling Variance Gamma model. In NIO, the high variance makes the original Variance Gamma model unable to follow the pattern, so the result of reducing the variance of the original Variance Gamma model makes the model better understand the return pattern in general.

CONCLUSIONS AND RECOMMENDATIONS

For the results of this research, the Gram Charlier Expansion Black Scholes formula can really adjust the abnormal skewness and kurtosis in the option pricing using the Black-Scholes formula. Besides that, the Importance Sampling Variance Gamma model and the Antithetic Variate Variance Gamma model are really able to reduce the resulting simulations variance so that it can produce more accurate option prices with market prices compared to the Variance Gamma model. In the end, all Variance Gamma models are able to produce a call option price that is more in line with the market price than all Black Scholes models when the log return data are not normally distributed. For the recommendations, we recommend the utilization of neural network architecture for the Black-Scholes model or for the Variance Gamma model so that the model can better capture patterns from the data.

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